



Economic-Mathematical Assessment and Scenario Analysis of Agricultural Enterprise Efficiency in Kazakhstan

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Abstract

Improving the efficiency of agricultural enterprises in Kazakhstan requires a comprehensive accounting of production, financial, technological, and risk factors, since an increase in nominal output does not always reflect a real improvement in performance. The aim of this study is to develop an economic-mathematical model for the quantitative assessment and forecasting of the effective development of agricultural enterprises in Kazakhstan amid a growing innovation-driven economy. The information base was compiled using official statistical data of Kazakhstan for 2010-2025 on gross agricultural output, volume indices, gross harvest and yield of grain and leguminous crops, the introduction of water-saving technologies, and technical modernization of the industry. The methodological framework combines indicator standardization, entropy weighting, an innovation- and risk-augmented production function, an optimization formulation for resource allocation, and scenario-based forecasting of an integral efficiency index. The results showed that in 2019-2025, nominal agricultural output increased by 88.5%, while the real physical volume increased by about 24%. In 2025, the yield of grain and leguminous crops reached 16.31 c/ha, exceeding the previous year's level by 7.2%. In the innovation scenario, the profit of a conditional enterprise increases by about 28.6%, and the return on costs increases from 53.85% to 67.22%. By 2030, the value of the integral index in a given innovation scenario reaches 140.26 points versus 110.41 points in the inertial scenario. The results obtained can be used in intra-company planning, evaluation of investment decisions, and monitoring the effectiveness of agricultural policy.

KEYWORDS

Agriculture, Agricultural Enterprise, Agricultural Economy, Agribusiness, Innovation-Driven Economy, Production Efficiency, Business Efficiency

1 | INTRODUCTION

Kazakhstan's agricultural sector is a geographically dispersed economic system in which food production, rural employment, export development, water security, and technological modernization are closely interconnected. According to the Bureau of National Statistics, the gross output of agriculture, forestry, and fisheries reached KZT 8,363.6 billion in 2024, while official government estimates indicate that gross agricultural output approached KZT 9.8 trillion in 2025 (Bureau of National Statistics, 2025). However, growth in nominal output alone is insufficient for assessing sectoral efficiency, as it may reflect price changes, favourable weather conditions, expanded cultivated area, or short-term resource exploitation.

For an agricultural enterprise, the quality of growth is more significant than its nominal magnitude: a sustainable increase in yield, a reduction in production costs, the rationalization of water consumption, an increase in technical readiness, a faster managerial response, and a reduction in uncertainty. Sustainable development requires higher yields, lower production costs, more efficient water use, improved technical readiness, faster managerial responses, and reduced uncertainty. In an innovation-driven economy, agricultural enterprises increasingly integrate field data, equipment monitoring, resource accounting, and financial information into a unified management system. Therefore, investments in digital technologies, water-saving systems, and machinery modernization should be assessed in terms of their measurable production and financial returns.

The problem addressed in this study is the absence of a unified applied model that integrates production, financial, technological, and risk-oriented indicators into a single forecasting system. In practice, isolated indicators dominate management: yield, profit, subsidies, sown area, gross harvest, investment, and debt. Each is important, but no single indicator can establish whether an enterprise is developing effectively. An increase in gross harvest, for instance, may be accompanied by rising water intensity, declining margins, and a deteriorating structure of working capital; conversely, a reduction in production costs may reflect insufficient fertilizer application and depress future yields. An economic-mathematical model is therefore required in which efficiency is defined as the dynamic integration of interrelated indicators and the forecast rests not on the mechanical extrapolation of the past but on a scenario-based assessment of the innovation cycle.

The aim of this study is to develop an economic-mathematical model for the quantitative assessment and forecasting of the effective development of agricultural enterprises in Kazakhstan amid a growing innovation-driven economy. To achieve this aim, the study systematizes theoretical approaches to modeling production efficiency; identifies variables relevant to the specific characteristics of the national agricultural sector; constructs an integrated index of effective development; presents a production-financial and optimization framework; performs calculation procedures based on open data and a hypothetical grain enterprise; and develops a scenario-based forecast through 2030. The scientific novelty lies in combining the production function, the entropy index, the optimization of resource allocation, and scenario forecasting within a single analytical framework

applicable to agricultural enterprises in Kazakhstan, and in validating the trend component of that framework against official physical-volume data.

2 | LITERATURE REVIEW

The theoretical foundation of this study draws on four complementary strands of the literature: production-function theory, the measurement of technical and allocative efficiency, forecasting methodology, and the applied economics of agricultural digitalization in Kazakhstan. Reviewing these strands together makes it possible to compare the methods, empirical designs, and principal findings on which existing studies rely; to identify the limitations they share; and to locate the specific research gap that the present model addresses.

The first strand concerns production functions, in which output is modelled as the result of interactions among labour, capital, and other production inputs. The classical Cobb–Douglas production function estimates output elasticities with respect to individual inputs and allows returns to scale to be assessed. Solow, in turn, interpreted technological change as a residual source of economic growth that cannot be explained solely by increases in labour and capital (Cobb & Douglas, 1928; Solow, 1957). A more flexible translog specification allows for interactions among inputs and variable production elasticities. Benedetti et al. (2019), for example, estimate a stochastic frontier production function using data from crop farms in Apulia, Italy. Their results reveal substantial differences in input-use efficiency across farms within the same region, with the greatest variation observed in fertilizer and irrigation use. This finding suggests that models based only on average production responses may conceal important farm-level heterogeneity. Laureti et al. (2021) extend this approach by incorporating a spatial lag into a stochastic frontier model to capture irrigation spillovers among neighbouring plots in Southern Italy. They demonstrate that water-use efficiency depends not only on individual farm decisions but also on the spatial organization of irrigation networks, a factor generally omitted from conventional production-function specifications.

These findings are highly relevant to Kazakhstan, where irrigated agriculture faces significant water constraints and agricultural regions differ substantially in resource availability and input-use conditions. Nevertheless, a stand-alone production-function approach primarily estimates the average or frontier response of output to production inputs and does not, by itself, integrate financial performance, risk, resource optimization, and forecasting into a unified decision-support framework.

The second strand addresses the measurement of efficiency and productivity. Data envelopment analysis, introduced by Charnes et al. (1978), evaluates the relative efficiency of comparable decision-making units using multiple inputs and outputs. Coelli et al. (2005) provide a broader methodological framework for distinguishing technical, allocative, and scale efficiency. This distinction is particularly important in agriculture: a farm may achieve high yields while maintaining an inefficient cost structure, use land productively but manage working capital poorly, or receive public support without generating sustainable investment re-

turns. Compared with parametric frontier methods, DEA does not require the prior specification of a production function. However, it is sensitive to measurement error and outliers and requires sufficiently comparable production units, while consistent enterprise-level data may be limited in Kazakhstan.

Recent empirical applications of stochastic frontier analysis allow a systematic cross-study comparison of efficiency levels, methodological choices, and the role of innovation and policy. Alemu et al. (2024), using a Cobb–Douglas stochastic frontier model for 35 sub-Saharan African countries over 2005–2020, report an average technical efficiency level of approximately 83 percent. Their findings indicate that labour and fertilizer inputs are positively associated with cereal production performance, whereas the expansion of cultivated land does not necessarily produce proportional efficiency gains. These results provide supporting evidence that agricultural growth cannot be explained solely by land expansion and that the productivity of variable inputs should also be considered.

Tleubayev et al. (2022) estimate a translog stochastic frontier model using farm-level data from 270 wheat producers in Kazakhstan and Russia. They obtain an average technical efficiency score of 74.4 percent, with almost one-fifth of farms operating below 60 percent efficiency. Their results indicate that fertilizer and other variable inputs have stronger output effects than land. They also find that managerial education, cooperative membership, and participation in insurance programmes are positively associated with efficiency, whereas subsidies show a negative association. These findings suggest that the effectiveness of agricultural support may depend not only on the amount of funding provided but also on its institutional design and the managerial capacity of recipients.

Parikoglou et al. (2024) employ a stochastic input-distance function for a panel of Irish dairy farms observed between 2008 and 2018. They construct a farm-level innovation index using k-means clustering of technology-adoption indicators and estimate that transition to a higher innovation state is associated with an average increase of 1.7 percentage points in technical efficiency. Their results also show that innovation affects individual production factors differently, indicating that its productivity effects are heterogeneous rather than uniform. The average efficiency score of approximately 90 percent nevertheless demonstrates that an efficiency gap remains even in a technologically advanced agricultural system.

Xie et al. (2024) analyse 443 small-scale mango producers in China using a digital agricultural technology service platform. They report positive associations between platform use, technical efficiency, and farm income, with improved information access and more precise input application acting as important transmission channels. Taken together, these studies suggest that agricultural efficiency depends not only on land scale and other conventional inputs but also on technology adoption, managerial capacity, institutional arrangements, and the quality of resource allocation. They further indicate that innovation should be represented through measurable adoption indicators and factor-specific channels rather than treated as a uniform technological shift.

To aggregate heterogeneous indicators into a composite index, the entropy

weight method is widely used to derive data-driven weights based on the degree of variation in individual indicators. Chen et al. (2023) compare several improved entropy-weighting methods in constructing a regional high-quality development index for Qinghai, China. Sun et al. (2023) combine entropy weighting with the G1 method and TOPSIS to assess the sustainable development of fresh agricultural product logistics enterprises. These studies justify the entropy-based weighting adopted here, but they also share a common limitation: entropy weighting is applied to static ranking or cross-sectional benchmarking rather than to a forward-looking production-financial model, so the resulting index describes the present state without projecting a development trajectory or embedding it in an optimization framework.

The third strand concerns forecasting methods and crop modelling. Regression models are commonly used to estimate statistical relationships between production outcomes and their determinants and to assess the significance of the corresponding coefficients. Contemporary forecasting practice additionally emphasizes out-of-sample accuracy, model stability, and scenario-based interpretation, particularly when agricultural data are affected by seasonality, structural change, climatic variability, and external shocks (Gujarati & Porter, 2009; Hyndman & Athanasopoulos, 2021). Gavasso-Rita et al. (2024), in a review of 192 studies applying crop simulation models such as DSSAT, APSIM, and AquaCrop to food-security assessments, conclude that process-based models represent biophysical mechanisms effectively but have limited capacity to incorporate economic variables, farmer decision-making, and institutional constraints.

Nowak (2021), reviewing the adoption of precision agriculture technologies in developed countries, shows that GPS guidance, variable-rate application, and yield mapping have achieved moderate levels of adoption, whereas integrated decision-support systems combining agronomic, economic, and managerial information remain less widespread. This evidence indicates that the connection between field-level technologies and enterprise-level financial planning remains insufficiently developed. Li et al. (2024) integrate the APSIM crop simulation model with machine-learning methods and report improved predictive accuracy for dryland wheat yields compared with a standalone machine-learning approach. Their findings demonstrate the potential benefits of combining complementary modelling methods. In a broader methodological sense, the present study follows this integrative principle by linking a production function with financial indicators, resource constraints, and scenario analysis rather than relying exclusively on statistical trend extrapolation.

The fourth strand examines Kazakhstan's transition from an extensive agricultural development model towards a more innovation-oriented approach. Mukhamedov et al. (2021) consider the development of the agro-industrial complex as an important component of economic and food security, while Suieubayeva et al. (2022) identify structural and institutional constraints affecting the sustainability of the agricultural sector. Asrepov et al. (2023) report a statistically significant relationship between innovation and the development of Kazakhstan's

agro-industrial complex. Bekbossinova and Doszhan (2025) examine the associations of digitalisation and investment with agricultural development, whereas Tsapova et al. (2025) analyse sector-specific features and institutional barriers to digitalisation in the agro-industrial complex. Abuova et al. (2023) discuss the role of digital transformation in supporting the transition towards a green economy, and Tsoy and Nurbatsin (2024) demonstrate territorial heterogeneity in agricultural development through the identification of regional agro-hubs.

At the international level, the OECD (2025) emphasizes the importance of institutional arrangements and policy instruments in agricultural development, while the FAO (2026) highlights the interdependence of digital transformation, water management, and institutional capacity in Kazakhstan's agricultural sector. Nevertheless, the reviewed Kazakhstani studies focus primarily on sectoral trends, institutional conditions, digitalisation, and regional differences. Within this body of literature, limited attention has been given to an integrated enterprise-level framework that simultaneously combines production and financial indicators, innovation and risk factors, resource-allocation optimization, and validated scenario forecasting. The present study addresses this methodological gap.

Taken together, the reviewed studies treat production, efficiency measurement, forecasting, and digitalization largely in isolation. The principal methodological and empirical characteristics of the key international studies discussed above are compared below. Stochastic frontier models estimate static or panel efficiency scores but do not embed these scores in a forecasting or optimization framework: once the efficiency is measured, the analyst has no tool for projecting how it will evolve under alternative investment or policy scenarios (Alemu et al., 2024; Tleubayev et al., 2022; Benedetti et al., 2019; Laureti et al., 2021; Parikoglou et al., 2024). Crop models and hybrid forecast approaches capture biophysical dynamics but omit the financial, cost, and risk dimensions that determine enterprise viability (Gavasso-Rita et al., 2024; Li et al., 2024). Composite-index methods aggregate heterogeneous indicators into a single score but treat it as a static ranking rather than a trajectory (Chen et al., 2023; Sun et al., 2023). Studies on digitalization and innovation in agriculture confirm the relevance of technology adoption but stop short of integrating the innovation variable into a production function with explicit risk and water-use parameters (Xie et al., 2024; Nowak, 2021; Bekbossinova & Doszhan, 2025; Tsapova et al., 2025). None of the reviewed works provides a unified framework that simultaneously integrates indicator standardization, entropy weighting, an innovation- and risk-augmented production function, resource-allocation optimization, and scenario forecasting into one reproducible instrument whose forecast component is validated against official data. The present study addresses precisely this gap and, in doing so, distinguishes itself from the models above by uniting descriptive, optimizing, and predictive functions within a common notation applicable at enterprise, district, and regional levels.

3 | RESEARCH METHODS

The methodology rests on the systemic decomposition of an agricultural enterprise viewed as a dynamic production-financial system whose state at time t is described by the vector s_t . Management actions u_t comprise land allocation, resource procurement, investment in equipment, and the deployment of digital and water-saving solutions; external uncertainty is denoted by ξ_t and technological parameters by θ_t . The state transition is given in equation (1):

$$s_{t+1} = F(s_t, u_t, \theta_t, \xi_t), \quad t = 1, 2, \dots, T \quad (1)$$

where:

- s_t – the state vector of the enterprise;
- u_t – the vector of management actions;
- θ_t – technological parameters;
- ξ_t – external uncertainty;
- T – the planning horizon.

Incentive indicators (yield, profit, technical readiness, labour productivity, and innovation receptivity) are normalized directly, whereas disincentive indicators (production costs, water consumption, equipment depreciation, and projected risk) are reverse-normalized using equation (2):

$$z_{ij} = \frac{x_{ij} - \min(x_j)}{\max(x_j) - \min(x_j)} \quad (2)$$

where:

- z_{ij} – the normalized value of indicator j for object i ;
- $\min_i x_j$ and $\max_i x_j$ – the minimum and maximum of indicator j across objects.

The integral index of effective development is calculated as a weighted sum of the normalized indicators. The entropy method is used to derive data-driven weights and reduce the influence of subjective judgement using equation (3):

$$E_i = \sum_{j=1}^m w_j z_{ij}, \quad \sum_{j=1}^m w_j = 1, \quad w_j \geq 0. \quad (3)$$

The entropy value, degree of diversification, and corresponding indicator weight are calculated as follows (4):

$$e_j = -\frac{1}{\ln n} \sum_{i=1}^n p_{ij} \ln p_{ij}, \quad d_j = 1 - e_j, \quad w_j = \frac{d_j}{\sum_{j=1}^m d_j} \quad (4)$$

where:

- E_i – the integral efficiency index of object i ;
- w_j – the weight of indicator j ;
- e_j – the entropy of indicator j ;
- d_j – the degree of diversification;

p_{ij} – the share of object i in the total value of indicator j .

The production-financial component of the proposed framework is represented by an innovation- and risk-augmented production function is (5):

$$Y_t = A_0 L_t^\alpha K_t^\beta M_t^\gamma W_t^\delta \exp(\varphi A_t - \rho R_t) \quad (5)$$

where:

Y_t – the agricultural output in period t ;

L_t – land in use;

K_t – capital and equipment;

M_t – material working capital;

W_t – water availability;

A_0 – baseline technological productivity;

A_t – the innovation receptivity;

R_t – the aggregate risk;

$\alpha, \beta, \gamma, \delta, \varphi, \rho$ – model parameters;

p_t – the selling price;

I_t – Innovation-period investment;

r – the discount rate.

The corresponding financial result is calculated as follows (6):

$$\Pi_t = p_t Y_t - C_t - I_t \quad (6)$$

where:

Π_t – total production costs;

I_t – innovation investment in period t ;

C_t – profit in period t .

The proposed framework also considers the resource constraints faced by agricultural enterprises, including the availability of land, working capital, water, investment resources, and acceptable levels of debt and production risk. In the present study, these constraints are incorporated conceptually rather than solved as a formal optimization problem, because comparable enterprise-level data are unavailable. A full resource-allocation model may be developed in future research using actual farm-level observations.

To illustrate the enterprise-level application of the proposed framework, the study assumes that the adoption of digital field management, precision fertilization, water-saving technologies, and upgraded equipment affects both production and costs. In the innovation scenario, crop yield is assumed to increase by 12%, while unit production cost is assumed to decrease by 8%. These values are treated as illustrative scenario assumptions rather than as econometrically estimated effects.

The potential effect of water-saving technology on water use per hectare is calculated as follows (7):

$$H_t^{new} = H_t^{base} (1 - \omega) \quad (7)$$

where:

H_t^{base} – the baseline water use per hectare;

H_t^{new} – the water use after technology adoption;

ω – the assumed water-saving rate.

The economic feasibility of the illustrative innovation package is evaluated using the net present value criterion (8):

$$NPV_I = \sum_{t=1}^T \frac{\Delta \Pi_t}{(1+r)^t} - I_0 \quad (8)$$

where:

NPV_I – the net present value of the innovation project;

$\Delta \Pi_t$ – the annual profit increment;

I_0 – the initial investment.

Finally, the illustrative trajectory of the integral efficiency index is generated separately for each scenario (9):

$$E_{t+1} = E_t (1 + g + \mu_A + \mu_B + \mu_D - \mu_R), \quad s \in \{\text{inertial, moderate, innovative}\} \quad (9)$$

where:

E_{t+1} – the integral efficiency index in period t ;

g – the baseline growth rate;

μ_A, μ_B, μ_D – the contributions of digital readiness, water-saving equipment, and technical readiness;

μ_R – the aggregate risk contribution.

The predictive accuracy of the trend component is assessed using the mean absolute percentage error (MAPE). Because gross agricultural output measured in current prices combines changes in both prices and production volume, forecast accuracy is evaluated using a real output-volume index rather than the nominal output series. The real index is constructed by chaining the official annual physical-volume indices of agricultural output, with 2009 set equal to 100. A log-linear trend is estimated using data for 2010–2023 and applied to generate out-of-sample forecasts for 2024 and 2025. MAPE is calculated as follows (10):

$$MAPE = \frac{100}{n} \sum_{i=1}^n \left| \frac{Y_i - \hat{Y}_i}{Y_i} \right| \quad (10)$$

where:

Y_i – the actual value;

$\hat{Y}_i$ – the forecast value;

n – the number of observations in the validation window.

The empirical base is formed on the principle of comparability of official statistical and institutional sources. Macro-sectoral indicators (gross output, physical-volume index, sown and harvested areas, gross harvests, and crop yields) are drawn from the Bureau of National Statistics; technological indicators (digitalization, water conservation, and the modernization of the technical base) are drawn from the responsible government agencies. The proposed framework is designed for application at the agricultural-enterprise level. However, because consistent panel data for individual enterprises are not available in open sources, the empirical illustration relies on sector-level statistics for Kazakhstan for 2010–2025. These data are used to establish the macro-sectoral context, calibrate selected benchmark values, and conduct a preliminary validation of the trend component. The enterprise-level application is illustrated using a hypothetical 10,000-hectare grain farm constructed from official sectoral yield data and explicitly stated price, cost, and innovation assumptions. Therefore, the calculations should be interpreted as an illustrative application of the proposed framework rather than as an econometric estimation based on a representative sample of agricultural enterprises.

Table 1 presents the complete list of model variables, their symbols, units of measurement, expected direction of effect, and data sources.

Table 1. Variables of the economic-mathematical model

Variable	Symbol	Unit	Effect	Data source
Output (gross)	Y	tons; million KZT	+	BNS
Land in use	L	hectares	+	BNS / enterprise
Capital and equipment	K	index; million KZT	+	enterprise
Material working capital	M	million KZT	+	enterprise
Water availability	W	m ³ / hectare	+	MWRI / enterprise
Innovation receptivity	A	index (0-1)	+	enterprise
Water-saving coefficient	B	share (0-1)	+	MWRI
Technical readiness	D	share (0-1)	+	enterprise
Digital observability	O	index (0-1)	+	enterprise
Unit production cost	c	KZT / ton	–	enterprise
Aggregate risk	R	index (0-1)	–	estimated
Equipment wear	-	percent	–	BNS / enterprise

MWRI - Ministry of Water Resources and Irrigation of the Republic of Kazakhstan.
“+” denotes an incentive, “–” a disincentive

Note: compiled by the authors

As shown in Table 1, the model incorporates twelve variables: output, land, capital, working capital, water availability, innovation receptivity, water-saving and technical-readiness coefficients, digital observability, unit production cost, aggregate risk, and equipment wear, each with a defined measurement unit and data source ensuring reproducibility.

Figure 1 summarizes the logical structure of the model.

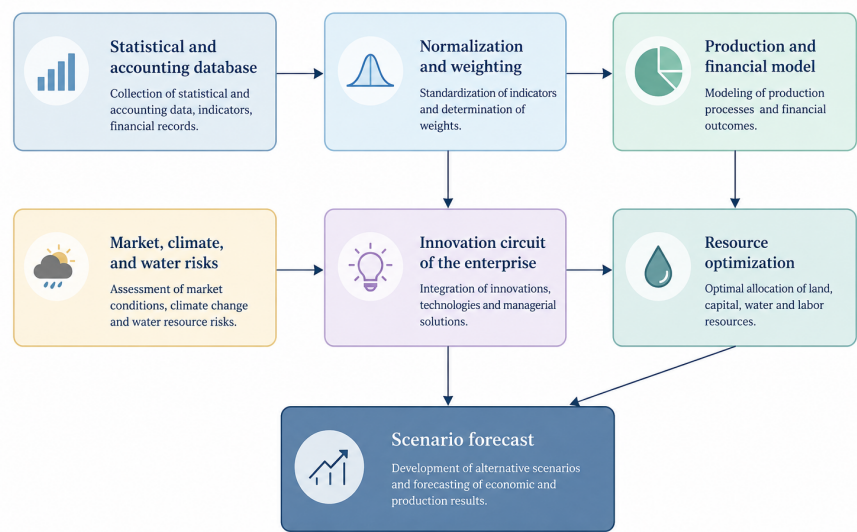


Figure 1. Block diagram of the economic-mathematical model of enterprise development

The block diagram shows that statistical and accounting data enter the model as inputs, pass through standardization and entropy weighting, feed into the production-financial model and optimization module, are bounded by the risk module, and produce the integral efficiency index and scenario forecast. A feed-back loop connects management decisions back to input data, reflecting the iterative nature of the planning cycle.

4 | RESULTS

The first result is that, for Kazakhstan, the efficiency of an agricultural enterprise should be measured not by a single financial indicator but by a combination of production, financial, technological, and risk-oriented indicators. Recent statistics reveal both positive and limiting trends: the sector shows growth in gross output, higher yields of cereals and legumes, the expansion of digital information systems, and the adoption of water-saving technologies, yet high weather dependence, water stress in the south, equipment wear, regional heterogeneity, and the weak integration of data into management decisions constrain the straightforward use of linear forecasts. Within the proposed framework, statistical and accounting data are inputs rather than the goal of the analysis: indicators are first standardized and weighted, production and financial models then link resources to outcomes, and the optimization and risk modules select and bound the allocation of land, capital, working capital, and innovation investment (Figure 1).

Trends in gross output confirm that agriculture remains one of the most economically significant sectors of the national economy. On the basis of official figures of 5.2 trillion tenge in 2019 and 9.8 trillion tenge in 2025, nominal output rose by 88.5 percent over the period. However, part of this increase reflects prices rather than physical volume. Chaining the official physical-volume indices into a real output-volume index (2009 = 100) shows that real volume grew by approximately 24 percent over the same 2019–2025 window. The divergence between the two trajectories is presented in Figure 2.

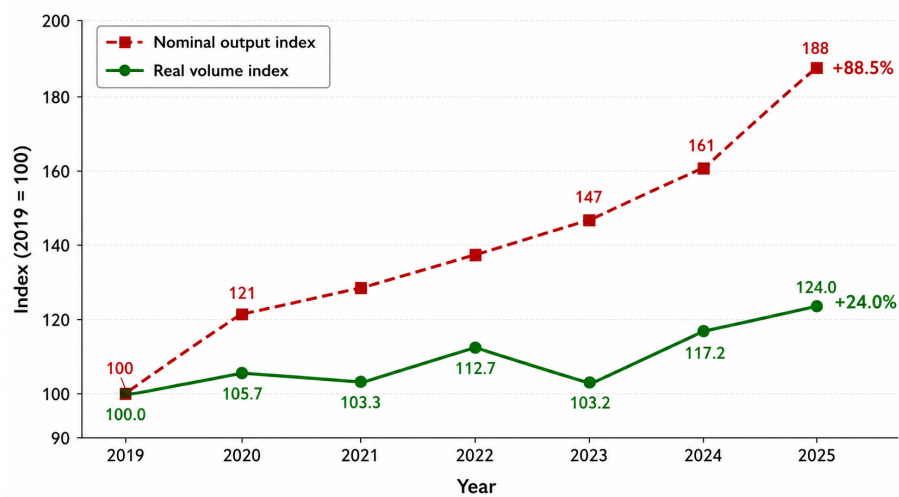


Figure 2. Nominal versus real gross agricultural output for 2019-2025 (2019 = 100)

Figure 2 shows that nominal agricultural output increased by 88.5% between 2019 and 2025, while the real output-volume index grew by approximately 24%. This divergence supports the central argument of the study: agricultural efficiency should not be assessed solely on the basis of nominal output, because nominal growth may overstate improvements in physical production when prices increase faster than output. Physical-volume, productivity, and cost indicators should therefore be considered jointly.

The following results concern crop production. In 2025, the gross harvest of cereals and legumes amounted to 25,946.3 thousand tonnes, while the harvested area totalled 15,907.4 thousand hectares (Bureau of National Statistics, 2025). Dividing gross harvest by harvested area yields an average productivity level of 1.631 tonnes per hectare, equivalent to 16.31 centners per hectare. According to the reported annual changes, yield was 7.2% higher than in 2024, gross harvest increased by 2.9%, and harvested area decreased by 3.9%. These rates imply approximate 2024 values of 15.21 centners per hectare, 25,215.06 thousand tonnes, and 16,552.97 thousand hectares, respectively. Because the published percentage changes are rounded, the reconstructed 2024 values may not be fully internally consistent. The

corresponding estimates are presented in Table 2.

Table 2. Estimated yields of cereals and legumes in Kazakhstan

Indicator	2024 (estimated)	2025	Change
Gross harvest, thousand tons	25,218.95	25,946.30	+2.9%
Harvested area, thousand ha	16,556.09	15,907.40	−3.9%
Yield, c/ha	15.21	16.31	+7.2%

Note: *compiled by the authors*

As Table 2 shows, the gross harvest rose by 2.9 percent while the harvested area decreased by 3.9 percent, resulting in a yield increase of 7.2 percent. This confirms that the growth in gross harvest was driven not by an expansion of cultivated area but by higher productivity per unit of area - a distinction fundamental to the model, where the land variable *L* must not dominate the technological variables *A*, *K*, *W*, and *M*.

Before generating forward-looking scenarios, the predictive accuracy of the trend component is assessed using the mean absolute percentage error. A log-linear trend was estimated for the real output-volume index using data for 2010–2023 and was then applied to generate out-of-sample forecasts for 2024 and 2025. The results are presented in Table 3. The in-sample MAPE is 3.6%, while the two-year out-of-sample MAPE is 4.6%. Both values are below the indicative benchmark of 10%; however, the out-of-sample result should be interpreted cautiously because the validation period contains only two annual observations. The largest in-sample deviation occurs in 2011, with an absolute percentage error of 10.75%, whereas the 2023 deviation is approximately 7.2%. These deviations illustrate the limitations of a deterministic trend in capturing short-term fluctuations and external shocks affecting agricultural production.

A separate set of results concerns water conservation. In 2024 water-saving technologies were implemented on 158,000 hectares, of which sprinkler systems accounted for 56,000 hectares, drip irrigation for 44,000 hectares, and land levelling (grading) for 58,000 hectares (Ministry of Water Resources and Irrigation of the Republic of Kazakhstan, 2025). The corresponding structural shares are 35.44 percent, 27.85 percent, and 36.71 percent. Applying (16) to a standard consumption of 6,000 m³/ha and a 30 percent saving yields a new consumption of 4,200 m³/ha and a total saving of about 284.4 million m³ on 158,000 hectares. The economic rationale of water conservation extends beyond reduced consumption: it lowers a farm’s dependence on water quotas, improves irrigation uniformity, reduces yield-loss risk, and stabilizes yield forecasting. When incorporated into the model, the water-saving coefficient *B_t* should therefore influence yield, cost, and risk simultaneously rather than being treated as a standalone technical indicator.

The structural breakdown of water-saving technology adoption is presented in Figure 3.

Table 3. Retrospective validation of the real volume index (2009 = 100)

Year	Actual	Forecast (trend)	APE, %
2010	89.60	94.63	5.61
2011	108.77	97.08	10.75
2012	92.78	99.60	7.35
2013	101.69	102.19	0.49
2014	102.71	104.84	2.08
2015	106.20	107.56	1.28
2016	111.94	110.35	1.41
2017	115.41	113.22	1.90
2018	119.44	116.16	2.75
2019	119.33	119.17	0.13
2020	126.13	122.26	3.06
2021	123.23	125.44	1.79
2022	134.44	128.69	4.27
2023	123.15	132.03	7.22
2024 (validation)	139.89	135.46	3.17
2025 (validation)	148.01	138.98	6.10

Note: compiled by the authors

Figure 3 shows a near-equal distribution across the three technology types, suggesting that no single solution dominates water-saving practice in Kazakhstan. The estimated aggregate water saving of 284.4 million cubic metres on 158 thousand hectares demonstrates the quantitative significance of the water-saving coefficient in the model and confirms that further expansion of irrigated area under water-saving technologies can materially reduce sectoral water stress.

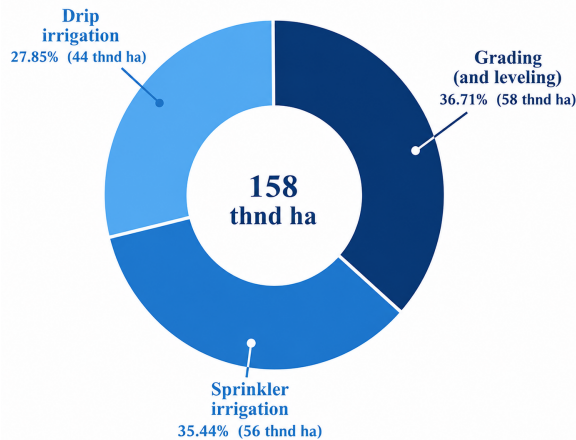


Figure 3. Structure of water-saving technology implementation in 2024

The framework is tested on a hypothetical grain enterprise of 10,000 hectares using the 2025 yield of 1.63 t/ha as the baseline, a selling price of 100,000 KZT/t, and a base cost of 65,000 KZT/t. In the baseline scenario output is 16,300 tons, revenue 1,630.0 million KZT, cost 1,059.5 million KZT, profit 570.5 million KZT, and cost profitability 53.85 percent. In the innovative scenario, digital field support, soil mapping, precision fertilization, and water-saving irrigation raise the yield by 12 percent and lower the unit cost by 8 percent, so that output rises to 18,256 tons, revenue to 1,825.6 million KZT, cost to 1,091.5 million KZT, and profit to 734.1 million KZT, lifting profitability to 67.26 percent. Table 4 summarizes the comparison. Profit grows faster than physical output (by 28.7 percent against 12 percent) because the enterprise simultaneously increases volume and reduces unit cost, which confirms that the innovation variable must enter both the output and the cost functions of the model.

Table 4. Baseline versus innovative scenario for a 10,000-hectare grain farm

Indicator	Baseline	Innovative	Change
Yield, t/ha	1.6300	1.8256	+12.0%
Gross output, tons	16,300	18,256	+1,956 t
Unit cost, KZT/t	65,000	59,800	−8.0%
Revenue, million KZT	1,630.0	1,825.6	+195.6
Cost, million KZT	1,059.5	1,091.5	+32.0
Profit, million KZT	570.5	734.1	+163.6
Cost profitability, %	53.85	67.26	+13.41 p.p.

Note: compiled by the authors

The economic feasibility of the innovation package is assessed with the net-present-value rule in (18). For an initial investment of 500.0 million KZT, an annual profit increment of 163.6 million KZT, a discount rate of 12 percent, and a five-year horizon, the annuity factor is 3.6048 and $NPV_I = 163.6 \times 3.6048 - 500.0 \approx 89.8$ million KZT. The positive value indicates that the package is economically justified under the stated parameters, while a higher interest rate, more expensive equipment, or a smaller profit increment could move the project into the negative zone. An enterprise therefore requires not a prescriptive rule on adopting innovations but a computational framework for evaluating each project.

For the sector a range of scenarios is preferable to a single forecast, because output depends on climate, prices, water, technology, financing, and institutional support. With the integral index set at 100 in 2025, the recursion (19) generates three trajectories: an inertial scenario (2 percent average annual growth), a moderately innovative scenario (5 percent), and an innovative scenario (7 percent). The numerical values are reported in Table 5.

By 2030, the innovative scenario reaches an index value of 140.26, compared with 127.63 for the moderately innovative and 110.41 for the inertial trajectory. The resulting gap of 29.85 index points translates into lower production costs, reduced water stress, and a more sustainable investment position.

Table 5. Scenario forecast of the integral efficiency index for 2025-2030

Year	Inertial (+2%)	Moderate (+5%)	Innovative (+7%)
2025	100.00	100.00	100.00
2026	102.00	105.00	107.00
2027	104.04	110.25	114.49
2028	106.12	115.76	122.50
2029	108.24	121.55	131.08
2030	110.41	127.63	140.26

Index set at 100 in 2025; $\Delta E(2030) = 140.26 - 110.41 = 29.85$ index points.

Note: compiled by the authors

The divergence of the three trajectories over time is visualized in Figure 4.

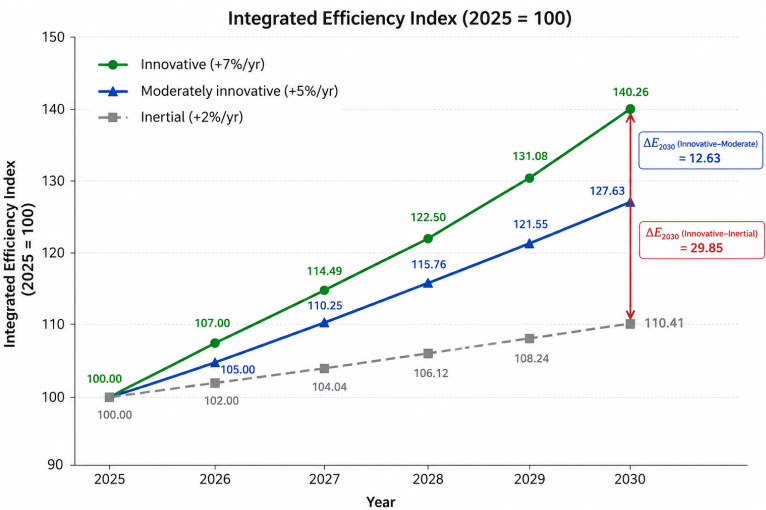


Figure 4. Scenario forecast of the integral efficiency index of an agricultural enterprise

Figure 4 illustrates that the trajectories begin to separate noticeably from 2027 onward, with the innovative scenario accelerating while the inertial path flattens. This pattern reflects the cumulative nature of the innovation effect: early investments in digitalization, water-saving technologies, and equipment renewal compound over time, producing returns that widen the gap with each successive year.

The relative influence of individual factors is examined through the sensitivity measure (20). The strongest disincentives are production costs, equipment wear, and water uncertainty, while the strongest incentives are crop yield, technical readiness, digital observability, and access to working capital; land exerts the weakest positive effect. This ordering is consistent with the state concept for the

development of the agro-industrial complex for 2021–2030 (Government of the Republic of Kazakhstan, 2021). Empirical estimation of the sensitivity coefficients from enterprise-level panel data is identified as a direction for further research.

5 | DISCUSSION

The practical implementation of the model proceeds in four phases. The diagnostic phase inventories land, equipment, water quotas, staffing, debt burden, cost structure, and the level of digital record-keeping. The calculation phase standardizes indicators, determines weights, computes the integral index, estimates the production function, and verifies the forecast error. The optimization phase compares alternative allocations of land, resources, investment, and water quotas. The management phase translates the chosen strategy into a production schedule, a cash-flow budget, an equipment-maintenance plan, a risk map, and a system of performance indicators.

Within this framework the distinction between extensive and intensive growth is decisive. If the sum of the elasticities of land, capital, and labour exceeds one, the enterprise operates under increasing returns to scale; but this state is unstable when it is achieved solely through land expansion and input procurement without improved technological coordination. When a substantial part of growth is explained by the term φA_t , the enterprise shifts to an intensive path in which the result is determined by the quality of resource integration rather than by the quantity of resources, which is the essence of the innovation economy.

The model is applicable at three levels. At the enterprise level it identifies which investments - fleet renewal, digital accounting, water conservation, seed stock, or logistics - yield the highest return. At the district level it compares enterprises and reveals binding constraints on efficiency. At the regional level it assesses how state-support measures affect actual technological outcomes rather than merely the volume of allocated funds. This multi-level applicability matters for Kazakhstan, whose agricultural zones are heterogeneous in climate, specialization, infrastructure, water access, and markets (Tsoy & Nurbatsin, 2024; OECD, 2025).

The model nonetheless has limitations. It requires a high-quality database: incomplete cost records, untracked water use, the absence of digital field maps, or costs that are not broken down by crop will bias the parameters. Separating the effect of innovation from weather requires panel data spanning several years. Market prices and resource costs may change faster than technological parameters, increasing forecasting uncertainty. Finally, the model does not replace agronomic expertise; it translates agronomic and financial decisions into a formal language but does not eliminate the need for professional interpretation.

Regional adaptation is therefore built into the approach. For northern grain regions the key variables are yield, the technical readiness of harvesting equipment, storage losses, transport costs, and export-related price risk; for southern irrigated regions, water constraints, irrigation efficiency, crop structure, the energy intensity of pumping, and the reliability of water-distribution channels; for suburban horticultural farms, cold-chain infrastructure, logistics speed, contract discipline, and

the share of processing. The framework does not impose a single set of weights but provides a common calculation structure within which weights and constraints are refined by natural-climatic zone, specialization, and market structure, in line with the international discussion on the digital transformation of the agro-industrial complex and the water sector (FAO, 2026).

For monitoring, a matrix of management thresholds is advisable. If the forecast error exceeds 15 percent, factor specifications and the quality of primary accounting should be reviewed; if the technical-readiness coefficient falls below a critical level, investment should prioritize repair and upgrading over expansion; if the normalized water-consumption ratio deteriorates as output grows, the growth should be regarded as conditionally unsustainable; and if the share of digitized operations rises without a fall in costs or downtime, the digital system performs an accounting rather than a management function. The final efficiency index should thus be accompanied not only by a numerical value but also by a diagnostic map showing which factor caused the deviation from the target trajectory.

The institutional dimension cannot be excluded. Subsidies, preferential lending, insurance, processing, and export infrastructure affect financial results through the cost of capital, the availability of working capital, and the reduction of market risk. Support should be evaluated not only by the volume of funds but by changes in productivity, profitability, water efficiency, and technological sustainability: if it raises current liquidity without lifting the integral index, its long-term effect is limited; if it stimulates fleet renewal, water conservation, digital accounting, and processing, it becomes part of an innovation multiplier. In this sense the model can serve as an analytical filter for agricultural policy, linking budgetary measures to final technological and economic outcomes, consistent with the development objectives of the agro-industrial complex through 2030 (Bekbossinova & Doszhan, 2025).

Econometric stability requires periodic re-estimation. If the same specification is retained while prices, climate, crop structure, lending rates, and support change, the coefficients gradually lose explanatory power; parameters should therefore be reassessed at least annually, with residual analysis and multicollinearity checks, and a transition to a panel specification is promising for enterprises with data on several fields and crops, because it separates technological effects from weather shocks and price changes. The principal value of the model lies less in the mechanical generation of forecast figures than in fostering a discipline of rational decision-making - defining constraints, measuring factors, comparing alternatives, and assessing consequences - which, for a sector marked by climatic uncertainty and spatial fragmentation, is a precondition for both profit growth and the sustainable development of rural areas.

6 | CONCLUSION

The study supports several conclusions. First, the effective development of agricultural enterprises in Kazakhstan within an innovation-driven economy should be treated as the dynamic interplay of interconnected production, financial, techno-

logical, and risk-oriented indicators; a simple increase in gross output or land area is not a sufficient measure of effectiveness. Second, the economic-mathematical model must combine indicator normalization, entropy-based weighting, an innovation- and risk-augmented production function, a resource-allocation optimization module, and scenario forecasting, so that managerial decisions are linked to measurable economic outcomes.

Third, calculations for cereals and legumes show that the growth of gross harvest in 2025 was associated with higher productivity despite a reduction in harvested area, which confirms the need to move from an extensive to a technology-intensive development path; the divergence between nominal (+88.5 percent) and real (+24 percent) growth over 2019-2025 reinforces this conclusion. Fourth, the trend component of the framework was validated against official physical-volume data, yielding a mean absolute percentage error of 4.6 percent out-of-sample, below the 10 percent adequacy threshold. Fifth, the hypothetical 10,000-hectare grain enterprise demonstrates that an innovation package can raise profit and profitability markedly through the simultaneous growth of yield and reduction of unit cost, and scenario forecasting reveals a gap of nearly 30 index points between the inertial and innovative trajectories by 2030.

The practical significance of the work lies in the use of the model for internal planning, regional monitoring, and the evaluation of state support: the framework identifies which factors - worn equipment, high production costs, weak digital observability, water uncertainty, or insufficient investment - constrain a given farm. Directions for further research include constructing a regional panel database, estimating production-function elasticities from actual enterprise data, applying spatial econometrics, and developing an applied decision-support system for agricultural management.

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